# Mark Scheme (Results)

# January 2007

GCE

## GCE Mathematics

## Core Mathematics C4 (6666)

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# January 2007
# 6666 Core Mathematics C4
# Mark Scheme

<table>
<tr><th>Question Number</th><th>Scheme</th><th></th><th>Marks</th></tr>
<tr><td></td><td>** represents a constant</td><td></td><td></td></tr>
<tr><td>1.</td><td>$\text{f}(x) = (2-5x)^{-2} = \underline{(2)^{-2}}\left(1-\frac{5x}{2}\right)^{-2} = \underline{\frac{1}{4}}\left(1-\frac{5x}{2}\right)^{-2}$</td><td>Takes 2 outside the bracket to give any of $(2)^{-2}$ or $\frac{1}{4}$.</td><td>B1</td></tr>
<tr><td></td><td>$= \frac{1}{4}\left\{\underline{1+(-2)(**x); + \frac{(-2)(-3)}{2!}(**x)^2 + \frac{(-2)(-3)(-4)}{3!}(**x)^3 + \ldots}\right\}$</td><td>Expands $(1+**x)^{-2}$ to give an unsimplified $1+(-2)(**x)$;<br><br>A correct unsimplified $\{\underline{\ldots\ldots\ldots}\}$ expansion with candidate's $(**x)$</td><td>M1<br><br>A1</td></tr>
<tr><td></td><td>$= \frac{1}{4}\left\{\underline{1+(-2)(\tfrac{-5x}{2}); + \frac{(-2)(-3)}{2!}(\tfrac{-5x}{2})^2 + \frac{(-2)(-3)(-4)}{3!}(\tfrac{-5x}{2})^3 + \ldots}\right\}$</td><td></td><td></td></tr>
<tr><td></td><td>$= \frac{1}{4}\left\{1 + 5x; + \frac{75x^2}{4} + \frac{125x^3}{2} + \ldots\right\}$</td><td></td><td></td></tr>
<tr><td></td><td>$= \frac{1}{4} + \frac{5x}{4}; + \frac{75x^2}{16} + \frac{125x^3}{8} + \ldots$</td><td>Anything that cancels to $\frac{1}{4} + \frac{5x}{4}$;<br>Simplified $\frac{75x^2}{16} + \frac{125x^3}{8}$</td><td>A1;<br>A1</td></tr>
<tr><td></td><td>$= \frac{1}{4} + 1\frac{1}{4}x; + 4\frac{11}{16}x^2 + 15\frac{5}{8}x^3 + \ldots$</td><td></td><td>[5]</td></tr>
<tr><td></td><td></td><td></td><td>5 marks</td></tr>
</table>

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** 1. **Way 2** | $f(x)=(2-5x)^{-2}$ | | |
| | | $\frac{1}{4}$ or $(2)^{-2}$ | B1 |
| | $=\left\{\underline{(2)^{-2}+(-2)(2)^{-3}(**x);+\frac{(-2)(-3)}{2!}(2)^{-4}(**x)^2+\frac{(-2)(-3)(-4)}{3!}(2)^{-5}(**x)^3+...}\right\}$ | Expands $(2-5x)^{-2}$ to give an unsimplifed $(2)^{-2}+(-2)(2)^{-3}(**x)$; | M1 |
| | | A correct unsimplified $\{\underline{.........}\}$ expansion with candidate's $(**x)$ | A1 |
| | $=\left\{(2)^{-2}+(-2)(2)^{-3}(-5x);+\frac{(-2)(-3)}{2!}(2)^{-4}(-5x)^2+\frac{(-2)(-3)(-4)}{3!}(2)^{-5}(-5x)^3+...\right\}$ | | |
| | $=\left\{\frac{1}{4}+(-2)(\frac{1}{8})(-5x);+(3)(\frac{1}{16})(25x^2)+(-4)(\frac{1}{16})(-125x^3)+...\right\}$ | | |
| | $=\frac{1}{4}+\frac{5x}{4};+\frac{75x^2}{16}+\frac{125x^3}{8}+...$ | Anything that cancels to $\frac{1}{4}+\frac{5x}{4}$; | A1; |
| | | Simplified $\frac{75x^2}{16}+\frac{125x^3}{8}$ | A1 |
| | $=\frac{1}{4}+1\frac{1}{4}x;+4\frac{11}{16}x^2+15\frac{5}{8}x^3+...$ | | |
| | | | **[5]** |
| | | | **5 marks** |

Attempts using Maclaurin expansions need to be referred to your team leader.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 2. (a) | Volume $= \pi \int_{-\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{3(1+2x)}\right)^2 dx = \frac{\pi}{9}\int_{-\frac{1}{4}}^{\frac{1}{2}} \frac{1}{(1+2x)^2} dx$ | **Use** of $V = \pi\int y^2\, dx$. Can be implied. Ignore limits. | B1 |
| | $= \left(\frac{\pi}{9}\right)\int_{-\frac{1}{4}}^{\frac{1}{2}} (1+2x)^{-2}\, dx$ | Moving their power to the top. **(Do not allow power of -1.)** Can be implied. Ignore limits and $\frac{\pi}{9}$ | M1 |
| | $= \left(\frac{\pi}{9}\right)\left[\frac{(1+2x)^{-1}}{(-1)(2)}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$ | Integrating to give $\pm p(1+2x)^{-1}$<br>$-\frac{1}{2}(1+2x)^{-1}$ | M1<br>A1 |
| | $= \left(\frac{\pi}{9}\right)\left[-\frac{1}{2}(1+2x)^{-1}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$ | | |
| | $= \left(\frac{\pi}{9}\right)\left[\left(\frac{-1}{2(2)}\right) - \left(\frac{-1}{2(\frac{1}{2})}\right)\right]$ | | |
| | $= \left(\frac{\pi}{9}\right)\left[-\frac{1}{4} - (-1)\right]$ | | |
| | $= \frac{\pi}{12}$ | Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef | A1 aef |
| | | | **[5]** |
| (b) | From Fig.1, AB $= \frac{1}{2} - \left(-\frac{1}{4}\right) = \frac{3}{4}$ units | | |
| | As $\frac{3}{4}$ units $\equiv$ 3cm | | |
| | then scale factor $k = \frac{3}{\left(\frac{3}{4}\right)} = 4$. | | |
| | Hence Volume of paperweight $= (4)^3\left(\frac{\pi}{12}\right)$ | $(4)^3 \times$ (their answer to part (a)) | M1 |
| | $V = \frac{16\pi}{3}\ \text{cm}^3 = 16.75516...\ \text{cm}^3$ | $\frac{16\pi}{3}$ or awrt 16.8 or $\frac{64\pi}{12}$ or aef | A1 |
| | | | **[2]** |
| | | | **7 marks** |

**Note**: $\frac{\pi}{9}$ (or implied) is not needed for the middle three marks of question 2(a).

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** 2. (a) **Way 2** | Volume $= \pi \int_{-\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{3(1+2x)}\right)^2 dx = \pi \int_{-\frac{1}{4}}^{\frac{1}{2}} \frac{1}{(3+6x)^2} dx$ | **Use** of $V = \pi \int y^2 \, dx$. Can be implied. Ignore limits. | B1 |
| | $= (\pi) \int_{-\frac{1}{4}}^{\frac{1}{2}} (3+6x)^{-2} \, dx$ | Moving their power to the top. **(Do not allow power of -1.)** Can be implied. Ignore limits and $\pi$ | M1 |
| | $= (\pi) \left[\frac{(3+6x)^{-1}}{(-1)(6)}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$ | Integrating to give $\pm p(3+6x)^{-1}$<br>$-\frac{1}{6}(3+6x)^{-1}$ | M1<br>A1 |
| | $= (\pi) \left[ -\frac{1}{6}(3+6x)^{-1} \right]_{-\frac{1}{4}}^{\frac{1}{2}}$ | | |
| | $= (\pi) \left[ \left(\frac{-1}{6(6)}\right) - \left(\frac{-1}{6(\frac{3}{2})}\right) \right]$ | | |
| | $= (\pi) \left[ -\frac{1}{36} - (-\frac{1}{9}) \right]$ | | |
| | $= \frac{\pi}{12}$ | Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef | A1 aef |
| | | | **[5]** |

**Note**: $\pi$ is not needed for the middle three marks of question 2(a).

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 3. (a) | $x = 7\cos t - \cos 7t\,,\ \ y = 7\sin t - \sin 7t\,,$ | | |
| | $\frac{dx}{dt} = -7\sin t + 7\sin 7t\,,\ \ \frac{dy}{dt} = 7\cos t - 7\cos 7t$ | Attempt to differentiate x **and** y with respect to t to give $\frac{dx}{dt}$ in the form $\pm A\sin t \pm B\sin 7t$<br>$\frac{dy}{dt}$ in the form $\pm C\cos t \pm D\cos 7t$ | M1 |
| | | Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$ | A1 |
| | $\therefore \frac{dy}{dx} = \frac{7\cos t - 7\cos 7t}{-7\sin t + 7\sin 7t}$ | Candidate's $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ | B1 $\sqrt{}$ |
| | | | **[3]** |
| (b) | When $t = \frac{\pi}{6}$, m(**T**) $= \frac{dy}{dx} = \frac{7\cos\frac{\pi}{6} - 7\cos\frac{7\pi}{6}}{-7\sin\frac{\pi}{6} + 7\sin\frac{7\pi}{6}}$; | Substitutes $t = \frac{\pi}{6}$ or $30^\circ$ into their $\frac{dy}{dx}$ expression; | M1 |
| | $= \underline{\frac{\frac{7\sqrt{3}}{2} - \left(-\frac{7\sqrt{3}}{2}\right)}{-\frac{7}{2} - \frac{7}{2}}} = \underline{\frac{7\sqrt{3}}{-7}} = \underline{-\sqrt{3}} = \underline{\text{awrt } -1.73}$ | to give any of the four underlined expressions oe **(must be correct solution only)** | A1 **cso** |
| | Hence m(**N**) $= \frac{-1}{-\sqrt{3}}$ or $\frac{1}{\sqrt{3}}$ = awrt 0.58 | Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient". | A1$\sqrt{}$ oe. |
| | When $t = \frac{\pi}{6}$,<br>$x = 7\cos\frac{\pi}{6} - \cos\frac{7\pi}{6} = \frac{7\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$<br>$y = 7\sin\frac{\pi}{6} - \sin\frac{7\pi}{6} = \frac{7}{2} - \left(-\frac{1}{2}\right) = \frac{8}{2} = 4$ | The point $\underline{\left(4\sqrt{3},\ 4\right)}$ or $\underline{(\text{awrt } 6.9,\ 4)}$ | B1 |
| | **N**: $y - 4 = \frac{1}{\sqrt{3}}\left(x - 4\sqrt{3}\right)$ | Finding an equation of a normal with their point and their normal gradient or finds c by using $y = (\text{their gradient})x + "c"$. | M1 |
| | **N**: $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$ | Correct simplified EXACT equation of <u>normal</u>. This is dependent on candidate using correct $\left(4\sqrt{3},\ 4\right)$ | <u>A1</u> oe |
| | or $4 = \frac{1}{\sqrt{3}}\left(4\sqrt{3}\right) + c \ \Rightarrow\ c = 4 - 4 = 0$ | | |
| | Hence **N**: $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$ | | |
| | | | **[6]**<br>**9 marks** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** 3. (a) **Way 2** | $x = 7\cos t - \cos 7t\,,\ \ y = 7\sin t - \sin 7t\,,$ | | |
| | $\frac{dx}{dt} = -7\sin t + 7\sin 7t\,,\ \ \frac{dy}{dt} = 7\cos t - 7\cos 7t$ | Attempt to differentiate x **and** y with respect to t to give $\frac{dx}{dt}$ in the form $\pm A\sin t \pm B\sin 7t$ $\frac{dy}{dt}$ in the form $\pm C\cos t \pm D\cos 7t$ | M1 |
| | | Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$ | A1 |
| | $\frac{dy}{dx} = \frac{7\cos t - 7\cos 7t}{-7\sin t + 7\sin 7t} = \frac{-7(-2\sin 4t\sin 3t)}{-7(2\cos 4t\sin 3t)} = \tan 4t$ | Candidate's $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ | B1 $\sqrt{}$ |
| | | | **[3]** |
| (b) | When $t = \frac{\pi}{6}$, m(**T**) $= \frac{dy}{dx} = \tan\frac{4\pi}{6}$; | Substitutes $t = \frac{\pi}{6}$ or $30^\circ$ into their $\frac{dy}{dx}$ expression; | M1 |
| | $= \frac{2\left(\frac{\sqrt{3}}{2}\right)(1)}{2\left(-\frac{1}{2}\right)(1)} = \underline{-\sqrt{3}} = \underline{\text{awrt } -1.73}$ | to give any of the three underlined expressions oe **(must be correct solution only)** | A1 **cso** |
| | Hence m(**N**) = $\frac{-1}{-\sqrt{3}}$ or $\frac{1}{\sqrt{3}}$ = awrt 0.58 | Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient". | A1$\sqrt{}$ oe. |
| | When $t = \frac{\pi}{6}$,<br>$x = 7\cos\frac{\pi}{6} - \cos\frac{7\pi}{6} = \frac{7\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$<br>$y = 7\sin\frac{\pi}{6} - \sin\frac{7\pi}{6} = \frac{7}{2} - \left(-\frac{1}{2}\right) = \frac{8}{2} = 4$ | The point $\underline{\left(4\sqrt{3},\ 4\right)}$ or $\underline{(\text{awrt } 6.9,\ 4)}$ | B1 |
| | **N**: $y - 4 = \frac{1}{\sqrt{3}}\left(x - 4\sqrt{3}\right)$ | Finding an equation of a normal with their point and their normal gradient or finds c by using $y = (\text{their gradient})x + \text{"c"}$. | M1 |
| | **N:** $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$ | Correct simplified EXACT equation of <u>normal</u>. This is dependent on candidate using correct $\left(4\sqrt{3},\ 4\right)$ | <u>A1</u> oe |
| | or $4 = \frac{1}{\sqrt{3}}\left(4\sqrt{3}\right) + c \Rightarrow c = 4 - 4 = 0$ | | |
| | Hence **N:** $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$ | | |
| | | | **[6]** |
| | | | **9 marks** |

**Beware:** A candidate finding an m(**T**) = 0 can obtain A1ft for m(**N**) $\to \infty$, but obtains M0 if they write $y - 4 = \infty(x - 4\sqrt{3})$. If they write, however, **N**: $x = 4\sqrt{3}$, then they can score M1.

**Beware:** A candidate finding an m(**T**) = $\infty$ can obtain A1ft for m(**N**) = 0, and also obtains M1 if they write $y - 4 = 0(x - 4\sqrt{3})$ or y = 4.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 4. (a) | $\frac{2x-1}{(x-1)(2x-3)} \equiv \frac{A}{(x-1)} + \frac{B}{(2x-3)}$ | | |
| | $2x-1 \equiv A(2x-3)+B(x-1)$ | Forming this identity. **NB**: A & B are not assigned in this question | M1 |
| | Let $x=\frac{3}{2}$, $2 = B\left(\frac{1}{2}\right) \Rightarrow B = 4$ | | |
| | Let $x=1$, $1 = A(-1) \Rightarrow A = -1$ | either one of $A=-1$ or $B=4$. both correct for their A, B. | A1 A1 |
| | giving $\frac{-1}{(x-1)} + \frac{4}{(2x-3)}$ | | **[3]** |
| (b) & (c) | $\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)}\,dx$ | Separates variables as shown Can be implied | B1 |
| | $= \int \frac{-1}{(x-1)} + \frac{4}{(2x-3)}\,dx$ | Replaces RHS with their partial fraction to be integrated. | M1 $\sqrt{}$ |
| | $\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + c$ | ***At least*** two terms in ln's ***At least*** two ln terms correct All three terms correct and '+ c' | M1 A1 $\sqrt{}$ A1 **[5]** |
| | $y=10,\ x=2$ gives $c=\ln 10$ | $c=\ln 10$ | B1 |
| | $\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + \ln 10$ | | |
| | $\ln y = -\ln(x-1) + \ln(2x-3)^2 + \ln 10$ | Using the power law for logarithms | M1 |
| | $\ln y = \ln\left(\frac{(2x-3)^2}{(x-1)}\right) + \ln 10$ or $\ln y = \ln\left(\frac{10(2x-3)^2}{(x-1)}\right)$ | Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c. | M1 |
| | $y = \frac{10(2x-3)^2}{(x-1)}$ | $y = \frac{10(2x-3)^2}{(x-1)}$ or aef. isw | A1 aef **[4]** |
| | | | **12 marks** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** 4. (b) & (c) **Way 2** | $\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)}\, dx$ | Separates variables as shown Can be implied | B1 |
| | $= \int \frac{-1}{(x-1)} + \frac{4}{(2x-3)}\, dx$ | Replaces RHS with their partial fraction to be integrated. | M1 $\sqrt{}$ |
| | $\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + c$ | ***At least*** two terms in ln's<br>***At least*** two ln terms correct<br>All three terms correct and '+ c' | M1<br>A1 $\sqrt{}$<br>A1 |
| | ***See below for the award of B1*** | ***decide to award B1 here!!*** | B1 |
| | $\ln y = -\ln(x-1) + \ln(2x-3)^2 + c$ | Using the power law for logarithms | M1 |
| | $\ln y = \ln\left(\frac{(2x-3)^2}{x-1}\right) + c$ | Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c. | M1 |
| | $\ln y = \ln\left(\frac{A(2x-3)^2}{x-1}\right)$ where $c = \ln A$ | | |
| | or $e^{\ln y} = e^{\ln\left(\frac{(2x-3)^2}{x-1}\right) + c} = e^{\ln\left(\frac{(2x-3)^2}{x-1}\right)} e^{c}$ | | |
| | $y = \frac{A(2x-3)^2}{(x-1)}$ | | |
| | $y = 10,\ x = 2$ gives $A = 10$ | $A = 10$ for B1 | award above |
| | $y = \frac{10(2x-3)^2}{(x-1)}$ | $y = \frac{10(2x-3)^2}{(x-1)}$ or aef & isw | A1 aef |
| | | | **[5] & [4]** |

**Note:** The B1 mark (part (c)) should be awarded in the same place on ePEN as in the Way 1 approach.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** (b) & (c) **Way 3** | $\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)}\,dx$ | Separates variables as shown Can be implied | B1 |
| | $= \int \frac{-1}{(x-1)} + \frac{2}{(x-\frac{3}{2})}\,dx$ | Replaces RHS with their partial fraction to be integrated. | M1 $\sqrt{}$ |
| | $\therefore \ln y = -\ln(x-1) + 2\ln(x-\frac{3}{2}) + c$ | ***At least*** two terms in ln's<br>***At least*** two ln terms correct<br>All three terms correct and '+ c' | M1<br>A1 $\sqrt{}$<br>A1<br>**[5]** |
| | $y = 10,\ x = 2$ gives $c = \underline{\ln 10 - 2\ln\left(\frac{1}{2}\right)} = \underline{\ln 40}$ | $c = \ln 10 - 2\ln\left(\frac{1}{2}\right)$ or $c = \ln 40$ | B1 oe |
| | $\therefore \ln y = -\ln(x-1) + 2\ln(x-\frac{3}{2}) + \ln 40$ | | |
| | $\ln y = -\ln(x-1) + \ln(x-\frac{3}{2})^2 + \ln 10$ | Using the power law for logarithms | M1 |
| | $\ln y = \ln\left(\frac{(x-\frac{3}{2})^2}{(x-1)}\right) + \ln 40$ or $\ln y = \ln\left(\frac{40(x-\frac{3}{2})^2}{(x-1)}\right)$ | Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c. | M1 |
| | $\underline{y = \frac{40(x-\frac{3}{2})^2}{(x-1)}}$ | $\underline{y = \frac{40(x-\frac{3}{2})^2}{(x-1)}}$ or aef. isw | A1 aef<br>**[4]** |

**Note:** Please mark parts (b) and (c) together for any of the three ways.

| Question Number | Scheme | Marks |
| --- | --- | --- |
| 5. (a) | $\sin x + \cos y = 0.5 \quad (\text{ eqn } *)$ | |
| | $\left\{\frac{dy}{dx} \times\right\}$ $\cos x - \sin y \frac{dy}{dx} = 0 \quad (\text{ eqn } \#)$<br>Differentiates implicitly to include $\pm \sin y \frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.) | M1 |
| | $\frac{dy}{dx} = \frac{\cos x}{\sin y}$<br>$\frac{\cos x}{\sin y}$ | A1 **cso** |
| | | **[2]** |
| (b) | $\frac{dy}{dx} = 0 \Rightarrow \frac{\cos x}{\sin y} = 0 \Rightarrow \cos x = 0$<br>Candidate realises that they need to solve 'their numerator' = 0 …or candidate sets $\frac{dy}{dx} = 0$ in their (eqn #) and attempts to solve the resulting equation. | M1 $\sqrt{}$ |
| | giving $x = -\frac{\pi}{2}$ or $x = \frac{\pi}{2}$<br>both $x = -\frac{\pi}{2}, \frac{\pi}{2}$ or $x = \pm 90^\circ$ or awrt $x = \pm 1.57$ required here | A1 |
| | When $x = -\frac{\pi}{2}$, $\sin\left(-\frac{\pi}{2}\right) + \cos y = 0.5$<br>When $x = \frac{\pi}{2}$, $\sin\left(\frac{\pi}{2}\right) + \cos y = 0.5$<br>Substitutes either their $x = \frac{\pi}{2}$ or $x = -\frac{\pi}{2}$ into eqn $*$ | M1 |
| | $\Rightarrow \cos y = 1.5 \Rightarrow$ y has no solutions<br>$\Rightarrow \cos y = -0.5 \Rightarrow y = \frac{2\pi}{3}$ or $-\frac{2\pi}{3}$<br>Only one of $y = \frac{2\pi}{3}$ or $-\frac{2\pi}{3}$ or $120^\circ$ or $-120^\circ$ or awrt -2.09 or awrt 2.09 | A1 |
| | In specified range $(x, y) = \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)$<br>Only exact coordinates of $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)$<br>**Do not award this mark if candidate states other coordinates inside the required range.** | A1 |
| | | **[5]** |
| | | **7 marks** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 6. | $y = 2^x = e^{x\ln 2}$ | | |
| (a) **Way 1** | $\frac{dy}{dx} = \ln 2 . e^{x\ln 2}$ | $\frac{dy}{dx} = \ln 2 . e^{x\ln 2}$ | M1 |
| | Hence $\frac{dy}{dx} = \ln 2 .(2^x) = 2^x \ln 2$ **AG** | $2^x \ln 2$ **AG** | A1 **cso** |
| | | | **[2]** |
| ***Aliter*** (a) **Way 2** | $\ln y = \ln\left(2^x\right)$ leads to $\ln y = x\ln 2$ | Takes logs of both sides, then uses the power law of logarithms… | |
| | $\frac{1}{y}\frac{dy}{dx} = \ln 2$ | … and differentiates implicitly to give $\frac{1}{y}\frac{dy}{dx} = \ln 2$ | M1 |
| | Hence $\frac{dy}{dx} = y\ln 2 = 2^x \ln 2$ **AG** | $2^x \ln 2$ **AG** | A1 **cso** |
| | | | **[2]** |
| (b) | $y = 2^{(x^2)} \Rightarrow \frac{dy}{dx} = 2x.\, 2^{(x^2)}.\ln 2$ | $Ax\ 2^{(x^2)}$ | M1 |
| | | $2x.\, 2^{(x^2)}.\ln 2$ or $2x.y.\ln 2$ if y is defined | A1 |
| | When x = 2, $\frac{dy}{dx} = 2(2)2^4 \ln 2$ | Substitutes x = 2 into their $\frac{dy}{dx}$ which is of the form $\pm k\, 2^{(x^2)}$ or $Ax\ 2^{(x^2)}$ | M1 |
| | $\frac{dy}{dx} = \underline{64\ln 2} = 44.3614...$ | $\underline{64\ln 2}$ or awrt 44.4 | A1 |
| | | | **[4]** |
| | | | **6 marks** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** 6. (b) **Way 2** | $\ln y = \ln\left(2^{x^2}\right)$ leads to $\ln y = x^2 \ln 2$ | | |
| | $\frac{1}{y}\frac{dy}{dx} = 2x.\ln 2$ | $\frac{1}{y}\frac{dy}{dx} = Ax.\ln 2$ | M1 |
| | | $\frac{1}{y}\frac{dy}{dx} = 2x.\ln 2$ | A1 |
| | When x = 2, $\frac{dy}{dx} = 2(2)2^4 \ln 2$ | Substitutes x = 2 into their $\frac{dy}{dx}$ which is of the form $\pm k\,2^{(x^2)}$ or $Ax\ 2^{(x^2)}$ | M1 |
| | $\frac{dy}{dx} = \underline{64\ln 2} = 44.3614...$ | $\underline{64\ln 2}$ or awrt 44.4 | A1 |
| | | | **[4]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 7. | $\mathbf{a} = \overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \quad \Rightarrow \lvert\overrightarrow{OA}\rvert = 3$<br>$\mathbf{b} = \overrightarrow{OB} = \mathbf{i} + \mathbf{j} - 4\mathbf{k} \quad \Rightarrow \lvert\overrightarrow{OB}\rvert = \sqrt{18}$<br>$\overrightarrow{BC} = \pm(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \Rightarrow \lvert\overrightarrow{BC}\rvert = 3$<br>$\overrightarrow{AC} = \pm(\mathbf{i} + \mathbf{j} - 4\mathbf{k}) \Rightarrow \lvert\overrightarrow{AC}\rvert = \sqrt{18}$ | | |
| (a) | $\mathbf{c} = \overrightarrow{OC} = \underline{3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}}$ | $\underline{3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}}$ | B1 **cao**<br>**[1]** |
| (b) | $\overrightarrow{OA} \bullet \overrightarrow{OB} = \begin{pmatrix}2\\2\\1\end{pmatrix} \bullet \begin{pmatrix}1\\1\\-4\end{pmatrix} = \underline{2+2-4} = 0$ or…<br>$\overrightarrow{BO} \bullet \overrightarrow{BC} = \begin{pmatrix}-1\\-1\\4\end{pmatrix} \bullet \begin{pmatrix}2\\2\\1\end{pmatrix} = \underline{-2-2+4} = 0$ or…<br>$\overrightarrow{AC} \bullet \overrightarrow{BC} = \begin{pmatrix}1\\1\\-4\end{pmatrix} \bullet \begin{pmatrix}2\\2\\1\end{pmatrix} = \underline{2+2-4} = 0$ or…<br>$\overrightarrow{AO} \bullet \overrightarrow{AC} = \begin{pmatrix}-2\\-2\\-1\end{pmatrix} \bullet \begin{pmatrix}1\\1\\-4\end{pmatrix} = \underline{-2-2+4} = 0$ | An attempt to take the dot product between either $\overrightarrow{OA}$ and $\overrightarrow{OB}$ $\overrightarrow{OA}$ and $\overrightarrow{AC}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$ or $\overrightarrow{OB}$ and $\overrightarrow{BC}$<br>Showing the result is equal to zero. | <u>M1</u><br>A1 |
| | and therefore OA is perpendicular to OB and hence OACB is a rectangle. | <u>perpendicular</u> and <u>OACB is a rectangle</u> | A1 **cso** |
| | Area $= 3 \times \sqrt{18} = 3\sqrt{18} = 9\sqrt{2}$ | Using distance formula to find either the correct height or width.<br>Multiplying the rectangle's height by its width.<br>exact value of $3\sqrt{18}$, $9\sqrt{2}$, $\sqrt{162}$ or aef | M1<br>M1<br>A1<br>**[6]** |
| (c) | $\overrightarrow{OD} = \mathbf{d} = \frac{1}{2}(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$ | $\underline{\frac{1}{2}(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})}$ | B1<br>**[1]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| (d)<br><br>**Way 1** | *using dot product formula*<br>$\overrightarrow{DA} = \pm\left(\tfrac{1}{2}\mathbf{i} + \tfrac{1}{2}\mathbf{j} + \tfrac{5}{2}\mathbf{k}\right)$ & $\overrightarrow{DC} = \pm\left(\tfrac{3}{2}\mathbf{i} + \tfrac{3}{2}\mathbf{j} - \tfrac{3}{2}\mathbf{k}\right)$<br>or $\overrightarrow{BA} = \pm(\mathbf{i} + \mathbf{j} + 5\mathbf{k})$ & $\overrightarrow{OC} = \pm(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$ | Identifies a set of two relevant vectors<br>Correct vectors ± | M1<br>A1 |
| | $\cos D = (\pm)\dfrac{\begin{pmatrix}0.5\\0.5\\2.5\end{pmatrix}\bullet\begin{pmatrix}1.5\\1.5\\-1.5\end{pmatrix}}{\frac{\sqrt{27}}{2}.\frac{\sqrt{27}}{2}} = (\pm)\dfrac{\frac{3}{4}+\frac{3}{4}-\frac{15}{4}}{\frac{27}{4}} = (\pm)\dfrac{1}{3}$ | Applies dot product formula on multiples of these vectors.<br>Correct ft. application of dot product formula | |
| | $D = \cos^{-1}\left(-\dfrac{1}{3}\right)$ | Attempts to find the correct angle D rather than $180^{\circ} - D$. | ddM1 √ |
| | $D = 109.47122...^{\circ}$ | $109.5^{\circ}$ or awrt $109^{\circ}$ or $1.91^{c}$ | A1 |
| ***Aliter*** | *using dot product formula and direction vectors* | | **[6]** |
| (d)<br><br>**Way 2** | $\mathbf{d}\overrightarrow{BA} = \pm(\mathbf{i} + \mathbf{j} + 5\mathbf{k})$ & $\mathbf{d}\overrightarrow{OC} = \pm(\mathbf{i} + \mathbf{j} - \mathbf{k})$ | Identifies a set of two direction vectors<br>Correct vectors ± | M1<br>A1 |
| | $\cos D = (\pm)\dfrac{\begin{pmatrix}1\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}1\\1\\5\end{pmatrix}}{\sqrt{3}.\sqrt{27}} = (\pm)\dfrac{1+1-5}{\sqrt{3}.\sqrt{27}} = (\pm)\dfrac{1}{3}$ | Applies dot product formula on multiples of these vectors.<br>Correct ft. application of dot product formula. | dM1<br>A1 √ |
| | $D = \cos^{-1}\left(-\dfrac{1}{3}\right)$ | Attempts to find the correct angle D rather than $180^{\circ} - D$. | ddM1 √ |
| | $D = 109.47122...^{\circ}$ | $109.5^{\circ}$ or awrt $109^{\circ}$ or $1.91^{c}$ | A1<br>**[6]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** (d) **Way 3** | *using dot product formula and similar triangles* | | |
| | $d\overrightarrow{OA} = (2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ & $d\overrightarrow{OC} = (\mathbf{i} + \mathbf{j} - \mathbf{k})$ | Identifies a set of two direction vectors<br>Correct vectors | M1<br>A1 |
| | $\cos\left(\frac{1}{2}D\right) = \dfrac{\begin{pmatrix}2\\2\\1\end{pmatrix}\bullet\begin{pmatrix}1\\1\\-1\end{pmatrix}}{\sqrt{9}.\sqrt{3}} = \underline{\dfrac{2+2-1}{\sqrt{9}.\sqrt{3}}} = \underline{\dfrac{1}{\sqrt{3}}}$ | Applies dot product formula on multiples of these vectors.<br><u>Correct ft. application of dot product formula.</u> | dM1<br>A1 $\sqrt{}$ |
| | $D = 2\cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ | Attempts to find the correct angle D by doubling their angle for $\frac{1}{2}D$ . | ddM1 $\sqrt{}$ |
| | $D = 109.47122...^{\circ}$ | $109.5^{\circ}$ or awrt $109^{\circ}$ or $1.91^{c}$ | A1 |
| | | | **[6]** |
| ***Aliter*** (d) **Way 4** | *using cosine rule*<br>$\overrightarrow{DA} = \frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{5}{2}\mathbf{k}$ , $\overrightarrow{DC} = \frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} - \frac{3}{2}\mathbf{k}$ , $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$ | | |
| | $\lvert\overrightarrow{DA}\rvert = \dfrac{\sqrt{27}}{2}$ , $\lvert\overrightarrow{DC}\rvert = \dfrac{\sqrt{27}}{2}$, $\lvert\overrightarrow{AC}\rvert = \sqrt{18}$ | Attempts to find all the lengths of all three edges of $\Delta$ ADC<br>All Correct | M1<br>A1 |
| | $\cos D = \underline{\dfrac{\left(\frac{\sqrt{27}}{2}\right)^2 + \left(\frac{\sqrt{27}}{2}\right)^2 - \left(\sqrt{18}\right)^2}{2\left(\frac{\sqrt{27}}{2}\right)\left(\frac{\sqrt{27}}{2}\right)}} = -\underline{\dfrac{1}{3}}$ | Using the cosine rule formula with correct 'subtraction'.<br><u>Correct ft application of the cosine rule formula</u> | dM1<br>A1 $\sqrt{}$ |
| | $D = \cos^{-1}\left(-\dfrac{1}{3}\right)$ | Attempts to find the correct angle D rather than $180^{\circ} - D$. | ddM1 $\sqrt{}$ |
| | $D = 109.47122...^{\circ}$ | $109.5^{\circ}$ or awrt $109^{\circ}$ or $1.91^{c}$ | A1 |
| | | | **[6]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** (d) **Way 5** | *using trigonometry on a right angled triangle* $\overrightarrow{DA} = \frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{5}{2}\mathbf{k}$ $\overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$ | | |
| | Let X be the midpoint of AC | Attempts to find two out of the three lengths in $\Delta ADX$ | M1 |
| | $\lvert\overrightarrow{DA}\rvert = \frac{\sqrt{27}}{2}$, $\lvert\overrightarrow{DX}\rvert = \frac{1}{2}\lvert\overrightarrow{OA}\rvert = \frac{3}{2}$, $\lvert\overrightarrow{AX}\rvert = \frac{1}{2}\lvert\overrightarrow{AC}\rvert = \frac{1}{2}\sqrt{18}$ (hypotenuse), (adjacent) , (opposite) | Any two correct | A1 |
| | $\sin(\frac{1}{2}D) = \frac{\frac{\sqrt{18}}{2}}{\frac{\sqrt{27}}{2}}$ , $\cos(\frac{1}{2}D) = \frac{\frac{3}{2}}{\frac{\sqrt{27}}{2}}$ or $\tan(\frac{1}{2}D) = \frac{\frac{\sqrt{18}}{2}}{\frac{3}{2}}$ | Uses correct sohcahtoa to find $\frac{1}{2}D$ | dM1 |
| | | Correct ft application of sohcahtoa | A1 $\sqrt{}$ |
| | eg. $D = 2\tan^{-1}\left(\frac{\frac{\sqrt{18}}{2}}{\frac{3}{2}}\right)$ | Attempts to find the correct angle D by doubling their angle for $\frac{1}{2}D$ . | ddM1 $\sqrt{}$ |
| | $D = 109.47122...^{\circ}$ | $109.5^{\circ}$ or awrt $109^{\circ}$ or $1.91^{c}$ | A1 |
| | | | **[6]** |
| ***Aliter*** (d) **Way 6** | *using trigonometry on a right angled similar triangle OAC* $\overrightarrow{OC} = 3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ $\overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$ | | |
| | $\lvert\overrightarrow{OC}\rvert = \sqrt{27}$ , $\lvert\overrightarrow{OA}\rvert = 3$, $\lvert\overrightarrow{AC}\rvert = \sqrt{18}$ (hypotenuse), (adjacent), (opposite) | Attempts to find two out of the three lengths in $\Delta OAC$ | M1 |
| | | Any two correct | A1 |
| | $\sin(\frac{1}{2}D) = \frac{\sqrt{18}}{\sqrt{27}}$ , $\cos(\frac{1}{2}D) = \frac{3}{\sqrt{27}}$ or $\tan(\frac{1}{2}D) = \frac{\sqrt{18}}{3}$ | Uses correct sohcahtoa to find $\frac{1}{2}D$ | dM1 |
| | | Correct ft application of sohcahtoa | A1 $\sqrt{}$ |
| | eg. $D = 2\tan^{-1}\left(\frac{\sqrt{18}}{3}\right)$ | Attempts to find the correct angle D by doubling their angle for $\frac{1}{2}D$ . | ddM1 $\sqrt{}$ |
| | $D = 109.47122...^{\circ}$ | $109.5^{\circ}$ or awrt $109^{\circ}$ or $1.91^{c}$ | A1 |
| | | | **[6]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| Aliter<br>7. (b) (i)<br><br>Way 2 | $\mathbf{c} = \overrightarrow{OC} = \pm(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$<br>$\overrightarrow{AB} = \pm(-\mathbf{i} - \mathbf{j} - 5\mathbf{k})$ | | |
| | $\lvert\overrightarrow{OC}\rvert = \sqrt{(3)^2 + (3)^2 + (-3)^2} = \sqrt{(1)^2 + (1)^2 + (-5)^2} = \lvert\overrightarrow{AB}\rvert$ | A complete method of proving that the diagonals are equal. | **M1** |
| | As $\lvert\overrightarrow{OC}\rvert = \lvert\overrightarrow{AB}\rvert = \sqrt{27}$ | Correct result. | **A1** |
| | then the diagonals are equal, and OACB is a rectangle. | diagonals are equal and OACB is a rectangle | **A1 cso**<br><br>**[3]** |
| | $\mathbf{a} = \overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \Rightarrow \lvert\overrightarrow{OA}\rvert = 3$<br>$\mathbf{b} = \overrightarrow{OB} = \mathbf{i} + \mathbf{j} - 4\mathbf{k} \Rightarrow \lvert\overrightarrow{OB}\rvert = \sqrt{18}$<br>$\overrightarrow{BC} = \pm(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \Rightarrow \lvert\overrightarrow{BC}\rvert = 3$<br>$\overrightarrow{AC} = \pm(\mathbf{i} + \mathbf{j} - 4\mathbf{k}) \Rightarrow \lvert\overrightarrow{AC}\rvert = \sqrt{18}$<br>$\mathbf{c} = \overrightarrow{OC} = \pm(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}) \Rightarrow \lvert\overrightarrow{OC}\rvert = \sqrt{27}$<br>$\overrightarrow{AB} = \pm(-\mathbf{i} - \mathbf{j} - 5\mathbf{k}) \Rightarrow \lvert\overrightarrow{AB}\rvert = \sqrt{27}$ | | |
| Aliter<br>7. (b) (i)<br><br>Way 3 | $(OA)^2 + (AC)^2 = (OC)^2$<br>or $(BC)^2 + (OB)^2 = (OC)^2$<br>or $(OA)^2 + (OB)^2 = (AB)^2$<br>or $(BC)^2 + (AC)^2 = (AB)^2$ or equivalent | | |
| | $\Rightarrow \underline{(3)^2 + (\sqrt{18})^2 = (\sqrt{27})^2}$ | A complete method of proving that Pythagoras holds using their values.<br>Correct result | **M1**<br><br>**A1** |
| | and therefore OA is perpendicular to OB<br>or AC is perpendicular to BC<br>and hence OACB is a rectangle. | perpendicular and OACB is a rectangle | **A1 cso**<br><br>**[3]**<br>**14marks** |

| Question Number | Scheme | Marks |
|---|---|---|
| 8. (a) | | |

| x | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| y | $e^1$ | $e^2$ | $e^{\sqrt{7}}$ | $e^{\sqrt{10}}$ | $e^{\sqrt{13}}$ | $e^4$ |
| or y | 2.71828… | 7.38906… | 14.09403… | 23.62434… | 36.80197… | 54.59815… |

| | Scheme | Marks |
|---|---|---|
| | Either $e^{\sqrt{7}}$, $e^{\sqrt{10}}$ and $e^{\sqrt{13}}$ or awrt 14.1, 23.6 and 36.8 or e to the power awrt 2.65, 3.16, 3.61 (or mixture of decimals and e's) | |
| | ***At least*** two correct | B1 |
| | All three correct | B1 |
| | | **[2]** |
| (b) | $I \approx \frac{1}{2} \times 1 ; \times \underline{\left\{ e^1 + 2\left(e^2 + e^{\sqrt{7}} + e^{\sqrt{10}} + e^{\sqrt{13}}\right) + e^4 \right\}}$ — Outside brackets $\frac{1}{2} \times 1$ | B1; |
| | For structure of trapezium rule $\{............\}$; | M1 $\sqrt{}$ |
| | $= \frac{1}{2} \times 221.1352227... = 110.5676113... = \underline{110.6}$ (4sf) — $\underline{110.6}$ | A1 **cao** |
| | | **[3]** |

**Beware:** In part (b) candidates can add up the individual trapezia:

(b) $I \approx \frac{1}{2}.1\left(\underline{e^1 + e^2}\right) + \frac{1}{2}.1\left(\underline{e^2 + e^{\sqrt{7}}}\right) + \frac{1}{2}.1\left(\underline{e^{\sqrt{7}} + e^{\sqrt{10}}}\right) + \frac{1}{2}.1\left(\underline{e^{\sqrt{10}} + e^{\sqrt{13}}}\right) + \frac{1}{2}.1\left(\underline{e^{\sqrt{13}} + e^4}\right)$

| Question Number | Scheme | | Marks |
|---|---|---|---|
| (c) | $t = (3x+1)^{\frac{1}{2}} \Rightarrow \dfrac{dt}{dx} = \tfrac{1}{2}.3.(3x+1)^{-\frac{1}{2}}$ | $A(3x+1)^{-\frac{1}{2}}$ or $t\dfrac{dt}{dx} = A$ | M1 |
| | … or $t^2 = 3x+1 \Rightarrow \underline{2t\dfrac{dt}{dx} = 3}$ | $\underline{\tfrac{3}{2}(3x+1)^{-\frac{1}{2}}}$ or $\underline{2t\dfrac{dt}{dx} = 3}$ | A1 |
| | so $\dfrac{dt}{dx} = \dfrac{3}{2.(3x+1)^{\frac{1}{2}}} = \dfrac{3}{2t} \Rightarrow \dfrac{dx}{dt} = \dfrac{2t}{3}$ | Candidate obtains either $\frac{dt}{dx}$ or $\frac{dx}{dt}$ in terms of t … | |
| | $\therefore I = \int e^{\sqrt{(3x+1)}}\ dx = \int e^t \dfrac{dx}{dt}.dt = \int e^t.\dfrac{2t}{3}.dt$ | … and moves on to substitute this into I to convert an integral wrt x to an integral wrt t. | dM1 |
| | $\therefore I = \underline{\int \tfrac{2}{3} t e^t\ dt}$ | $\underline{\int \tfrac{2}{3} t e^t}$ | A1 |
| | change limits:<br>when x = 0, t = 1 & when x = 5, t = 4 | changes limits $x \to t$ so that $0 \to 1$ and $5 \to 4$ | B1 |
| | Hence $I = \int_1^4 \tfrac{2}{3} te^t\ dt$ ; where $a = 1, b = 4, k = \tfrac{2}{3}$ | | |
| | | | **[5]** |
| (d) | $\left\{\begin{array}{l} u = t \Rightarrow \frac{du}{dt} = 1 \\ \frac{dv}{dt} = e^t \Rightarrow v = e^t \end{array}\right\}$ | Let k be any constant for the first three marks of this part. | |
| | $k\int t e^t\ dt = k\left(t e^t - \int e^t.1\,dt\right)$ | Use of 'integration by parts' formula in the correct direction. | M1 |
| | | Correct expression with a constant factor k. | A1 |
| | $= k\left(\underline{t e^t - e^t}\right) + c$ | <u>Correct integration</u> with/without a constant factor k | A1 |
| | $\therefore \int_1^4 \tfrac{2}{3} te^t\ dt = \dfrac{2}{3}\left\{\left(4e^4 - e^4\right) - \left(e^1 - e^1\right)\right\}$ | Substitutes their changed limits into the integrand and subtracts oe. | dM1 **oe** |
| | $= \tfrac{2}{3}(3e^4) = \underline{2e^4} = 109.1963...$ | either $2e^4$ or awrt 109.2 | A1 |
| | | | **[5]**<br>**15 marks** |

- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark
- ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.